ICWE 16
August 27-31, 2023
FLORENCE - ITALY
16th International Conference on Wind Engineering

A yawed wake model to predict the velocity distribution of curled wake cross-section for wind turbines

Guangyi LIU [1], Qingshan Yang [2]

[1] *School of Civil Engineering, Chongqing University, Chongqing, China, liugy1993@cqu.edu.cn*
[2] *School of Civil Engineering, Chongqing University, Chongqing, China, qshyang@cqu.edu.cn*

SUMMARY:
An analytical yawed wake model for wind turbines, which predicts the streamwise velocity of yawed wake through including the curled shape of cross-section, is proposed in this study. In which, the conservations of mass and momentum are applied and the self-similarity Gaussian shape is adopted to describe the spanwise distribution of the streamwise velocity deficit and the skew angle. Compared with the existed wake models, addition equations are raised to predict the spanwise displacement of maximum velocity deficit at the different heights. For investigating the effectiveness of proposed model, the numerical simulations under different yaw angles are carried out to model the wake of a yawed wind turbine and the numerical method is also validated with the wind tunnel measurements before that. Comparisons of model predictions with the numerical simulation results show that this analytical model can acceptably predict the velocity distribution with heights in the far wake of a yawed wind turbine. Because of its good accuracy and low cost, the present yawed wake model beneficial to the implementation of yaw angle control.

Keywords: yawed wake model, wind turbines, velocity distribution

1. INTRODUCTION

When wind turbines are clustered in a wind farm, the wake effects of wind turbine are responsible for significant power losses. The yaw angle control is a potential control strategy for mitigating the wake effects of the existing wind farms. Through fixing a yaw angle between the rotating plane of wind rotor with the incoming wind direction, the wake development trajectory of upstream turbine will be redirected. The downstream wind turbine can partly escape the wake effects of upstream wind turbine and experience a higher inflow wind velocity to obtain more power output. A yawed wake model, which accurately evaluates the deflected wake trajectory and the velocity distribution of a yawed wind turbine, is essential for applying yaw angle control strategies in actual wind farms. Under the yawed situation, the thrust of wind rotor will induce a spanwise component to provide the wake with the spanwise momentum, which leads to the curled deformation of wake cross-section, which is essential to been considered into the yawed wake model. In the existed yawed wake models (Bastankhah and Porté-Agel, 2016; Qian and Ishihara, 2018), the cross-section shapes are adopted as a circle shape or an elliptical shape. These assumptions for the cross-section shape mean that the wake center is located at the hub height and the deformation of wake with heights are same, i.e. maximum velocity deficits of different heights are at the same spanwise location. However, as mentioned before, due to the deformation of wake cross-section, the maximum velocity deficit with height do not exist at the same spanwise location. The assumption of circle shape or elliptical shape will overestimate the spanwise deformation of yawed wake exclude the hub height. It will seriously influence the prediction of velocity distribution and downwind wind turbine power output. Therefore, in the yawed model, some additional formulas are needed to be built, which describe the deformation of wake cross-section.

In the present paper, a yawed wake model is proposed to prediction the streamwise velocity distribution in the far wake of a yawed wind turbine. The derivation of yawed wake model is exhibited to predict the velocity distribution of yawed wake in Section 2. In Section 3, the numerical simulations under different yaw angles are carried out to investigating the values predicted by the proposed wake model. Finally, the main conclusions of this study are summarized in Section 4.

2. YAWED WAKE MODEL

When the wind rotor has a fixed yaw angle with the inflow wind direction, the streamwise component of thrust will induce the streamwise velocity deficit of air current through the wind rotor and the spanwise component of thrust will increase the spanwise velocity of air current. Fig. 1 gives the control volume sketch of yawed wake model in the horizontal direction.

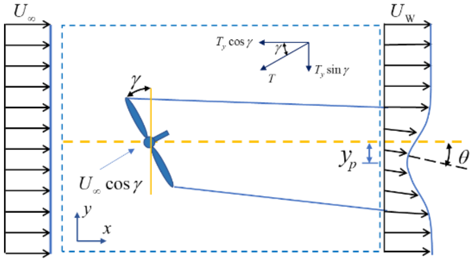


Fig. 1. Control volume around a yawed wind turbine

In Fig. 1, x and y are streamwise and spanwise coordinates, T_x and T_y are the streamwise and spanwise component of thrust respectively, U_∞ is the streamwise velocity of wake, U_∞ is the incoming velocity, θ is the skew angle of wake and y_p is wake deformation. Firstly, momentum equations have in the streamwise direction and spanwise direction respectively

$$0.5C_{\mathrm{T}}\rho A_0(U_\infty \cos\gamma)^2 \cos\gamma = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho U_{\mathrm{W}} \times \left(U_\infty - U_{\mathrm{W}}\right)\mathrm{d}y\mathrm{d}z \tag{1}$$

$$0.5C_{\mathrm{T}}\rho A_0(U_\infty \cos\gamma)^2 \sin\gamma = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho U_{\mathrm{W}} \times U_{\mathrm{W}} \tan\theta \mathrm{d}y\mathrm{d}z \tag{2}$$

According to the studies by Bastankhah and Porté-Agel (2016) and Qian and Ishihara (2018), the velocity distribution and the skew angel distribution of yawed wake exhibits some degree of self-similarity Gaussian type at the hub height. In addition, a linear expansion of wake width is referred to Bastankhah and Porté-Agel (2016). Through a series of algebraic manipulations, the wake velocity U_{W}, the wake deflection $y_{p,hub}$ and the skew angle θ have

$$U_{\mathrm{W}}(x,y,z) = U_\infty - \left(U_\infty - \sqrt{U_\infty^2 - \frac{1}{2}\frac{C_{\mathrm{T}} A_0 U_\infty^2 \cos^3\gamma}{\pi\sigma_y\sigma_z}} \right) e^{-\frac{\left(y - y_{\mathrm{p}}\right)^2}{2\sigma_y{}^2}} e^{-\frac{(z - z_{\mathrm{hub}})^2}{2\sigma_z{}^2}} \tag{3}$$

$$y_{p,hub} = \frac{D^2 C_{\mathrm{T}} \cos^2\gamma \sin\gamma}{32 n k_y k_z} \ln \frac{x - x_0 + m - n}{x - x_0 + m + n}\frac{m+n}{m-n} + y_{p0}, m = \frac{k_y \varepsilon_{z0} + k_z \varepsilon_{y0}}{2 k_y k_z}, n = \sqrt{m^2 - \frac{\varepsilon_{y0}\varepsilon_{z0}}{k_y k_z} + \frac{C_{\mathrm{T}} D^2 \cos^3\gamma}{20 k_y k_z}} \tag{4}$$

$$\theta = \frac{\frac{1}{2} C_{\mathrm{T}} \rho A_0 U_\infty^2 \cos^2 \gamma \sin \gamma}{2\pi \rho \sigma_y \sigma_z \left[U_\infty^2 - U_\infty C(x) e^{-\frac{1}{4}} + \frac{1}{3} C^2(x) e^{-\frac{1}{3}} \right]} e^{-\frac{\left(y\text{-}y_{\mathrm{p}} + \sigma_{\mathrm{y}}\right)^2}{2\sigma_y^{\,2}}} e^{-\frac{(z - z_{\mathrm{hub}})^2}{2\sigma_z^{\,2}}} \tag{5}$$

where, k_y and k_z are the wake expansion rate in the spanwise coordinates and vertical coordinates, σ_y and σ_z are the original widths of wake cross-section, x_0 is the original downwind location of far wake. Above the hub height, the spanwise displacement of maximum streamwise velocity deficit are almost uniform. While below the hub height, the spanwise displacement of maximum streamwise velocity deficit increases with height. Therefore, for convenience, a quadratic polynomial formula is adopted to predict the spanwise displacement of yawed wake under the hub height and the maximum value of this formula is existed at the hub height. As a result, the additional equations are given as

$$\begin{cases} y_p(z) = az^2 + bz + c, & z \le z_{\mathrm{hub}} \\ y_p(z) = y_{p,hub}, & z > z_{\mathrm{hub}} \end{cases} \tag{6}$$

where, $a = y_{p,hub} / (z_{\mathrm{hub}} - z_{\mathrm{bottom}})^2$, $b = -2y_{p,hub} z_{\mathrm{hub}} / (z_{\mathrm{hub}} - z_{\mathrm{bottom}})^2$ and $c = y_{p,hub} + y_{p,hub} z_{\mathrm{hub}}^2 / (z_{\mathrm{hub}} - z_{\mathrm{bottom}})^2$.

3. RESULTS AND DISCUSSIONS

A serious of CFD simulation, referred to the wind tunnel experiments (Chamorro and Porté-Agel, 2009), were carried out to validate the precision of proposed model. LES-ALM method provided by SOWFA was used to model wind turbine wake. For ensuring the reliability of numerical method, Fig. 2 shows comparisons of the mean streamwise velocity profiles and turbulence intensity profiles obtained from the simulations with the wind tunnel data. The present simulation results for wind turbine wake have been verified with high accuracy by comparison with the experimental data.

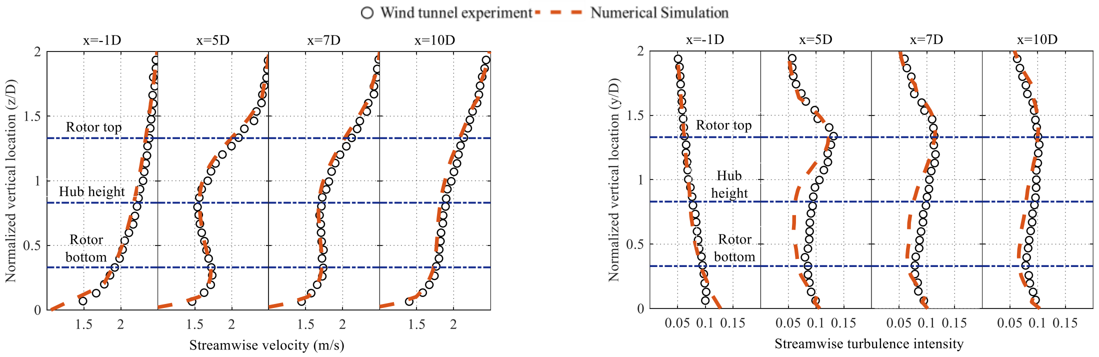
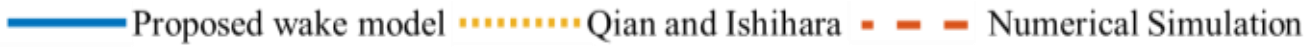
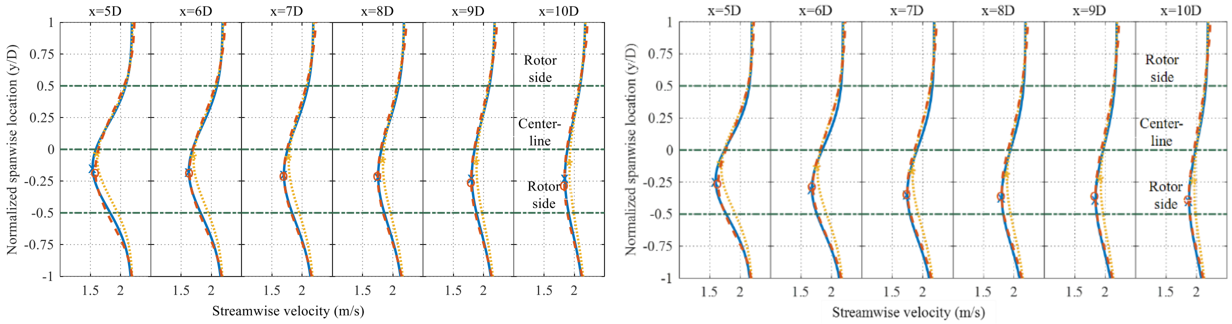


Figure 2. Comparisons between the wind tunnel experiment and the numerical simulation.

Fig. 3 gives the comparisons of wake velocity deficit between the simulation results, Qian and Ishihara (QI) and the predictions of proposed wake model are plotted at the hub height. It is evident that, compared with the numerical results, the proposed yawed model gives a similar prediction for the velocity profiles of wake and the spanwise displacement of wake center at the hub height. The maximum velocity deficit predicted by QI model is lower than the numerical results and the spanwise displacement of wake center is also underestimated.

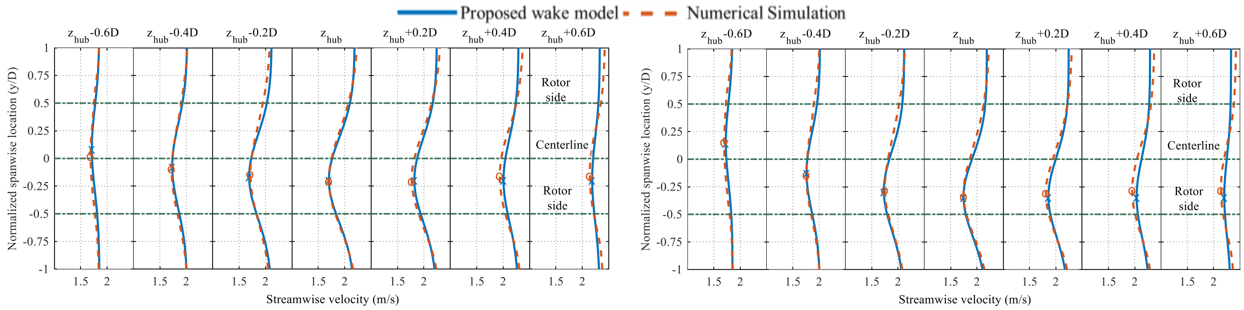




a) yaw angle=10 ° b) yaw angle=20 °

Figure 3. Validation between the wind tunnel experiment and the numerical simulation.

Fig. 4 gives the comparisons between the proposed yawed wake model and the numerical results with heights. In these figures, the horizontal profiles of the normalized mean velocity at selected downwind locations of $x = 7D$ and height locations $z = z_{\text{hub}} - 0.6D$, $z_{\text{hub}} - 0.4D$, $z_{\text{hub}} - 0.2D$, z_{hub}, $z_{\text{hub}} + 0.2D$, $z_{\text{hub}} + 0.4D$ and $z_{\text{hub}} + 0.6D$ are plotted. As Fig. 4 (a) and (b) illustrate, the proposed yawed model can not only give a similar distribution on the streamwise velocity, but also accurately predict the wake deformation at different heights.



a) x=7D, yaw angle=10 ° b) x=7D, yaw angle=20 °

Figure 4. Comparisons on spanwise profiles of the streamwise velocity with heights.

7. CONCLUSIONS

In this study, a yawed wake model for wind turbines is proposed and the deformation of wake cross-section is taken account into this model. For verifying the validation of proposed wake model, a series of numerical simulation were carried out by the LES-ALM and compared with the wind tunnel data to ensure the reliability of numerical method. The proposed yawed wake model is able to better predict the distribution of streamwise velocity with heights at the far wake region of a yawed wind turbine.

ACKNOWLEDGEMENTS

The work described in this paper was fully supported by grants from the 111 Project of the Ministry of Education and the Bureau of Foreign Experts of China (No. B18062) and Chongqing Science and Technology Bureau (cstc2020yszx-jcyjX0007). The authors thank the reviewers for their valuable comments and suggestions

REFERENCES

Bastankhah, M. and Porté-Agel, F., 2016. Experimental and theoretical study of wind turbine wakes in yawed conditions. Journal of Fluid Mechanics 806, 506-541.

Qian, G. and Ishihara, T., 2018. A New Analytical Wake Model for Yawed Wind Turbines. Energies 11(3), 665.

Chamorro, L. and Porté-Agel, F., 2009. A wind-tunnel investigation of wind-turbine wakes: boundary-layer turbulence effects. Boundary-Layer Meteorol 1321, 129–149.